Power Analysis

A working definition:

The **power** of a statistical test is the probability that the test will reject the null hypothesis when the alternative hypothesis is true (i.e., the probability of <u>not</u> committing a <u>type II error</u>)

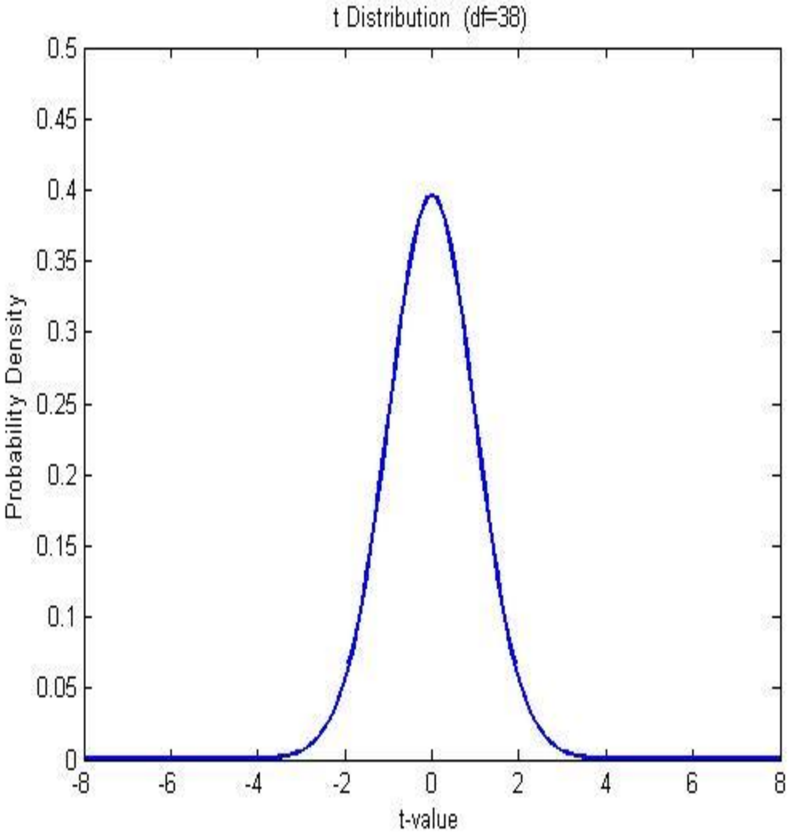
The **t statistic** is essentially the relative difference between two sample means

When those samples are drawn from the same population, we can accurately compute the probability of observing any/all values of **t** **(i.e., when H_0 is assumed to be true)**

This fact is realized as the <u>sampling distribution</u> of the **t** - whose shape is dependent upon a single parameter (i.e., the degrees of freedom imposed by the sampling and computational process…($df = n_1 - 1 + n_2 - 1 = n_1 + n_2 - 2$)

<u>MATLAB</u>:

```
t = [-6:0.1:6]; p = tpdf(t, 38); plot(t,p);
```



Sampling Distribution of t(38)

Type I Error Control: Setting Alpha

The experimenter can control the *a priori* probability of making a **type I error** (i.e., the erroneous rejection of the hypothesis-of-no-difference-H_0)

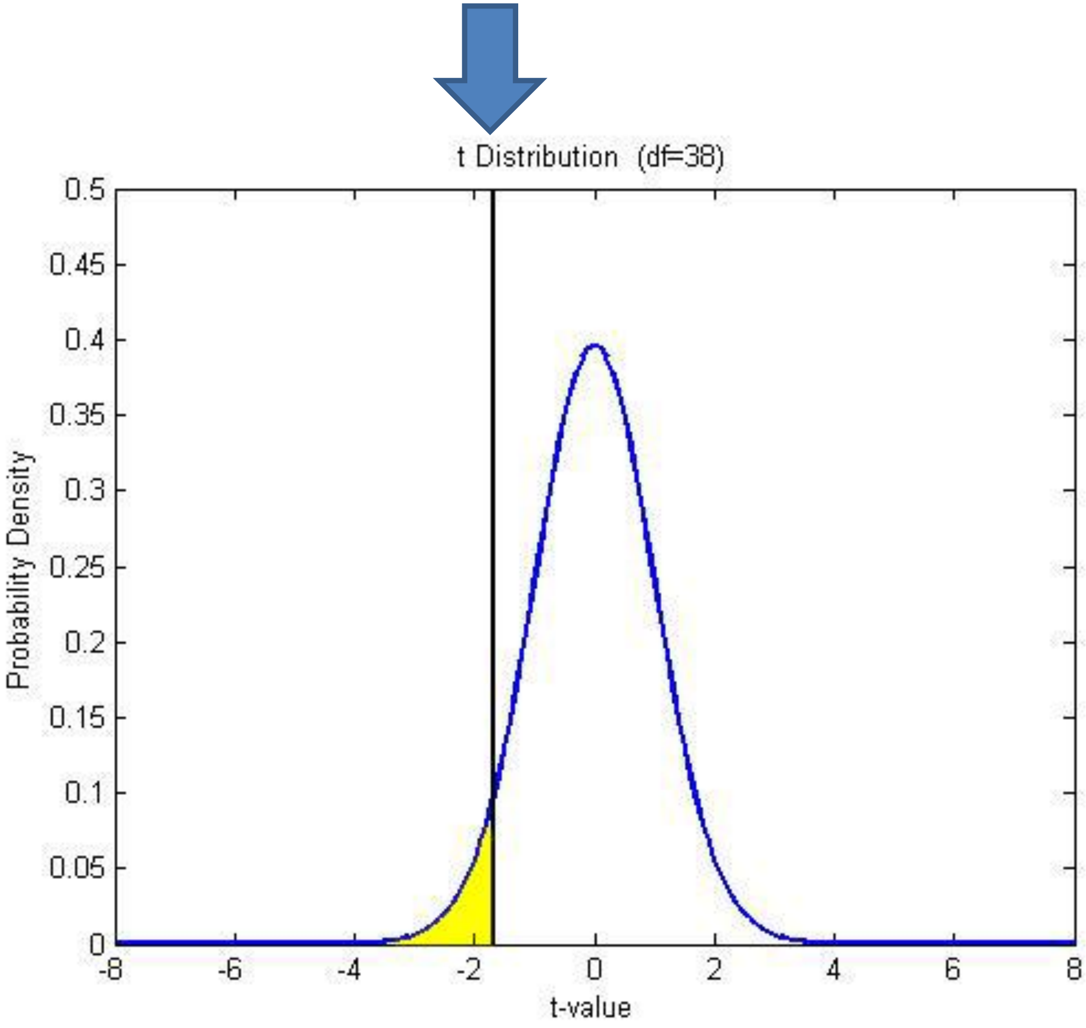
This is done by setting alpha.

Alpha sets the probability of observing a **FALSE POSITIVE**.

By convention, we set **α = 5%**
In the field of psychology.

MATLAB:
t_critical = tinv(0.05, 38);

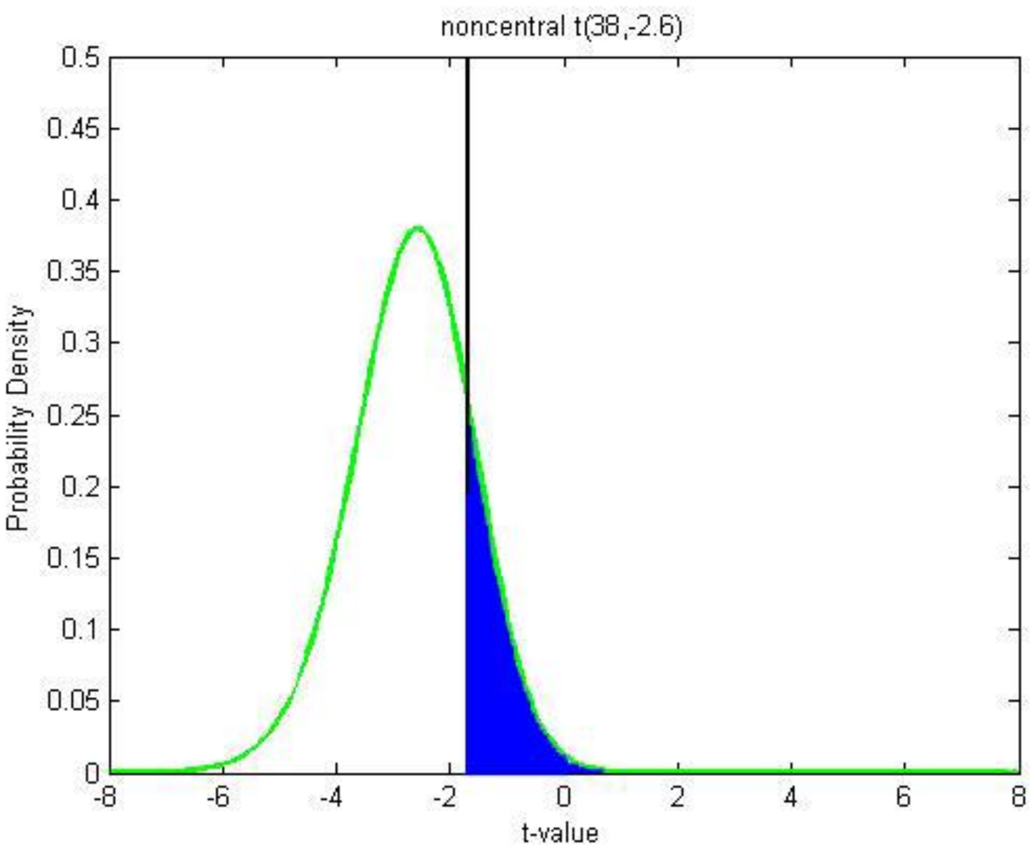
Decision Criterion:
t < 1.68



Q: How can we derive a measure of power from the familiar sampling distribution of the *t* statistic?

A: We can't. We need to consider how the *t* statistic is distributed when the alternative hypothesis is true. (i.e., the **noncentral t distribution**)

Noncentral t distribution
nct(df, δ)



The *noncentral t* represents a model of the t statistic when the alternative hypothesis is assumed to be true.

It has two parameters:

df (i.e., degrees of freedom)

δ (noncentrality parameter – delta)

Delta can most easily be described as a derivative of **"effect size"**:

Cohen's *d* = $(M_1-M_2)/((SD_1+SD_2)/2)$

$\delta = d * \sqrt{((n_1*n_2)/(n_1+n_2))}$

MATLAB:

```
t = [-8:0.1:8];
nct = nctpdf(t, df, delta);
plot(t, nct);
```

Let’s explore how the concept of power emerges from contrasting the **t** and **noncentral t** distributions

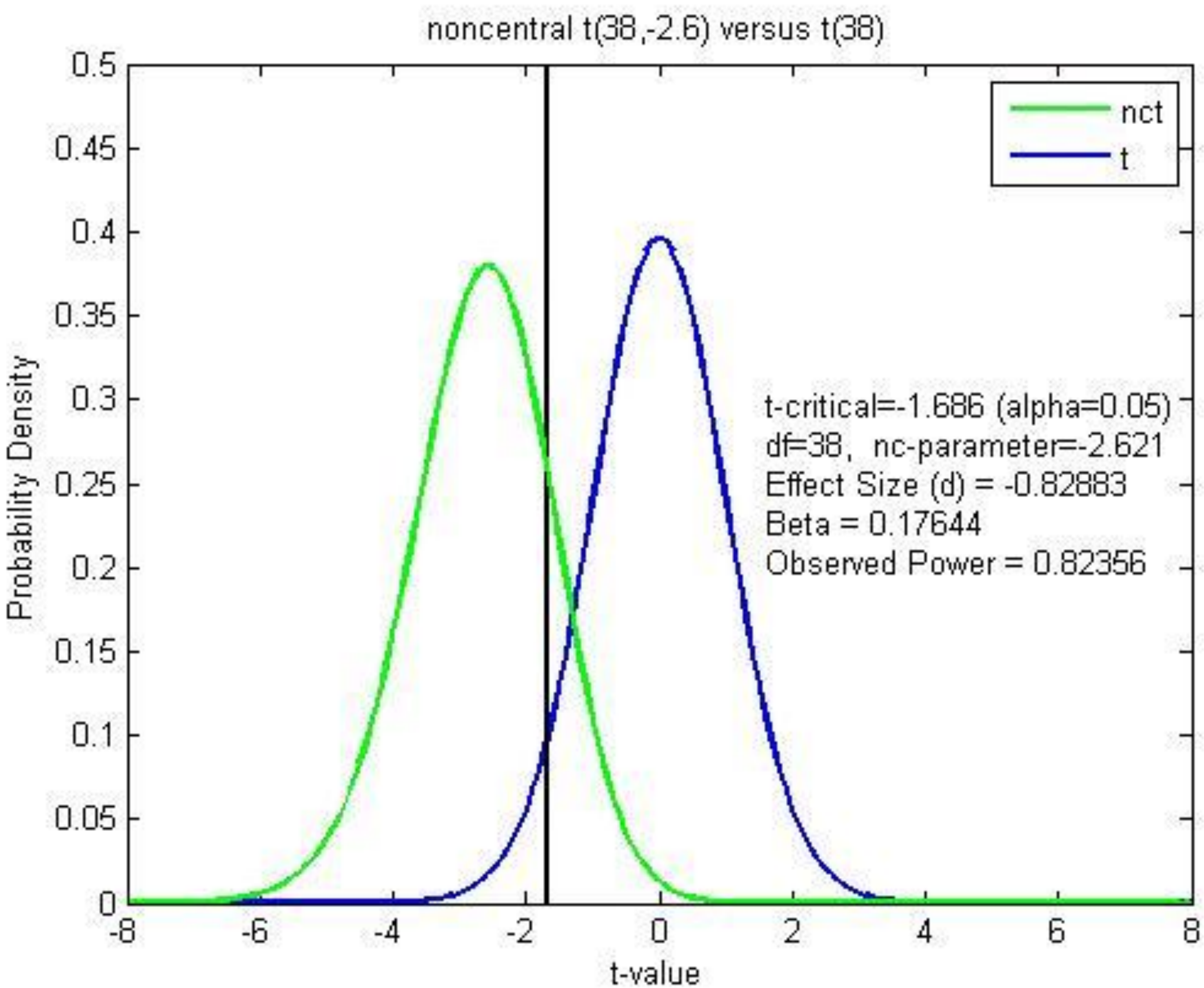
To make the following exercise more concrete, we'll focus on a specific example from a real study:

H_0: Treatment Group Mean RT = Control Group RT
H_1: Treatment Group Mean RT < Control Group Mean RT
(1-tailed t-test; alpha = 0.05)

<u>Control Group</u>:
$Mean_C = 1018$
$SD_C = 192$
$N_C = 20$

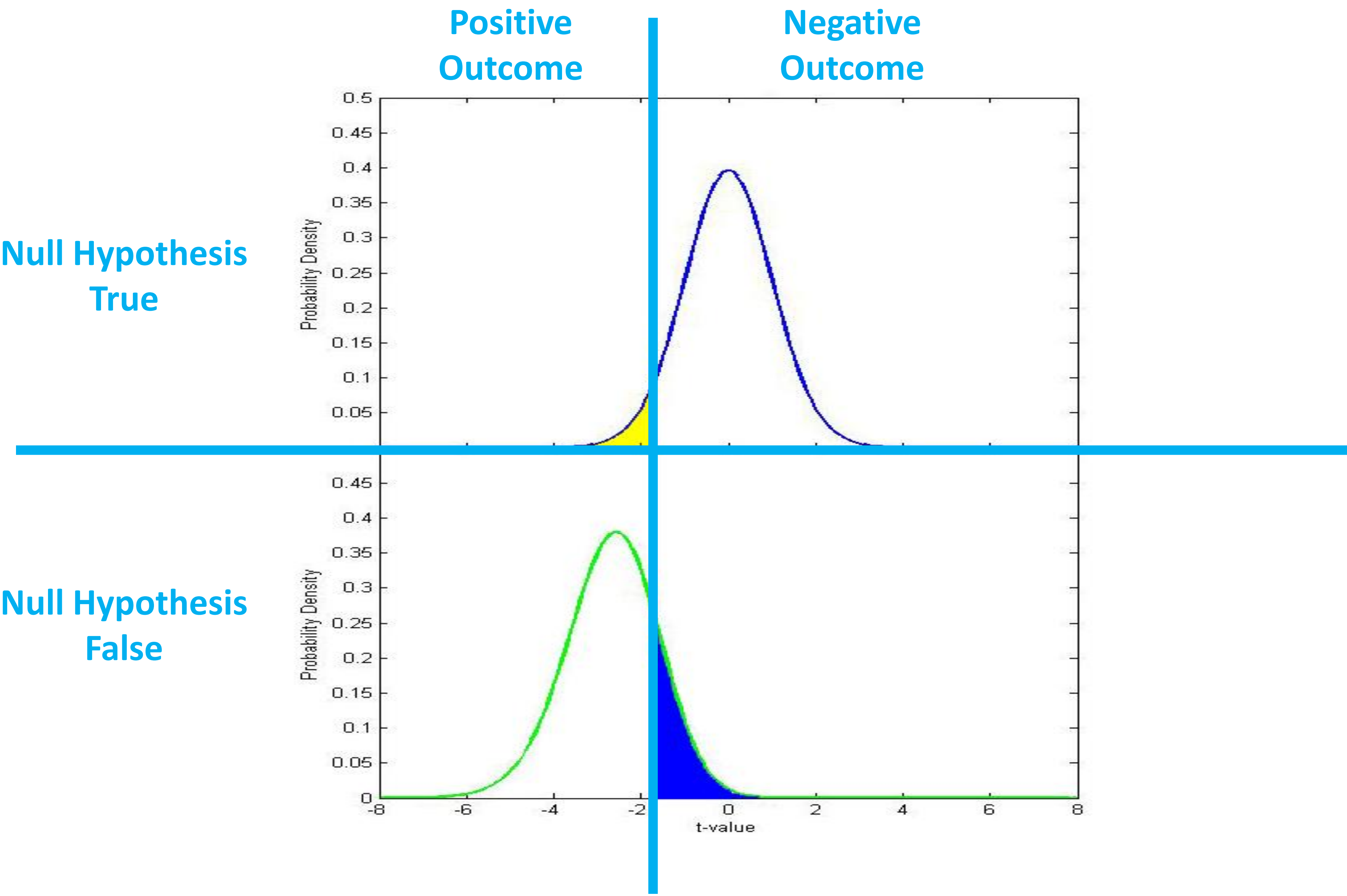
<u>Treatment Group</u>:
$Mean_T = 880$
$SD_T = 141$
$N_T = 20$

t versus noncentral t

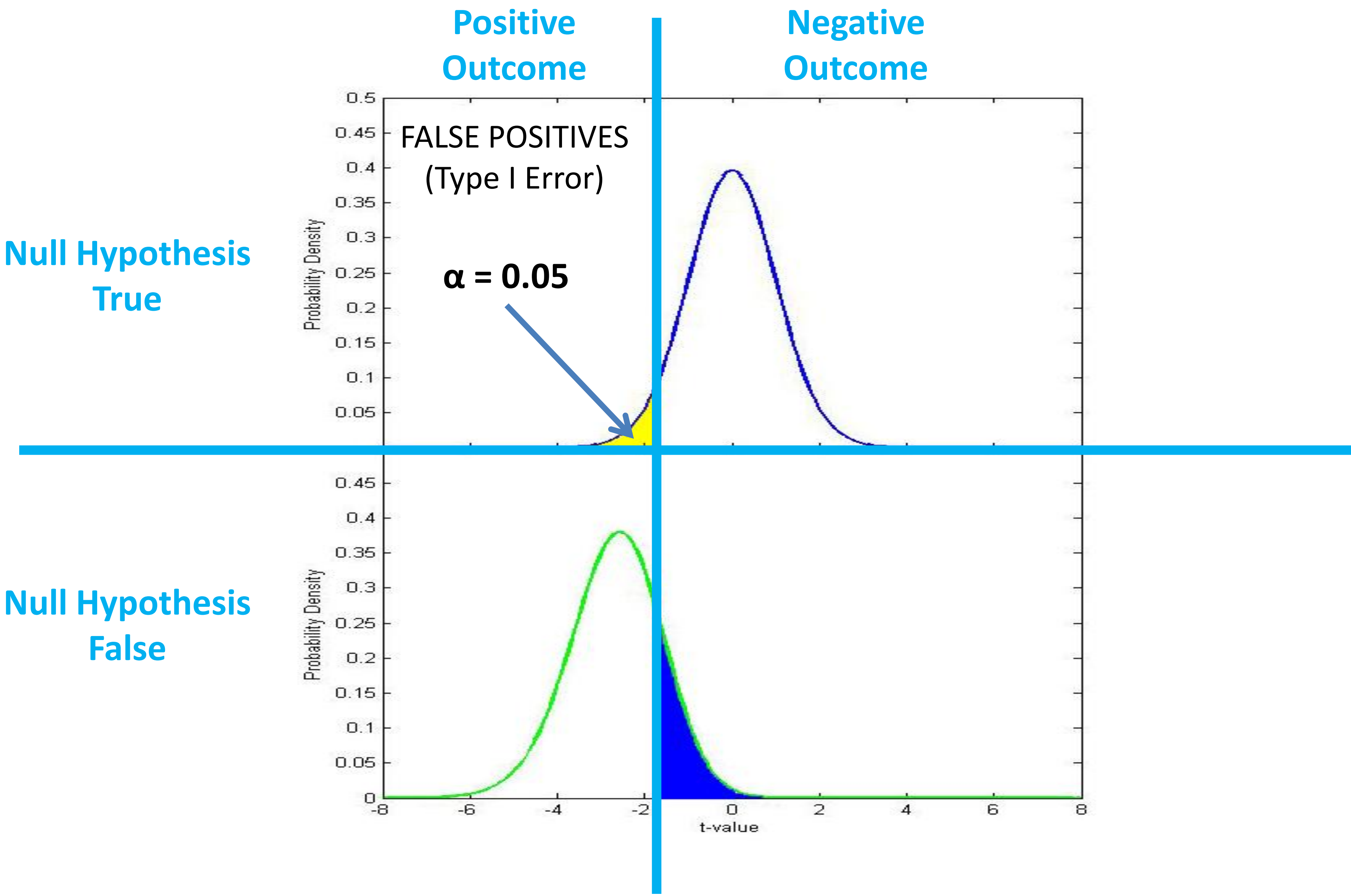


Let's consider these distributions from a Signal Detection Theory perspective

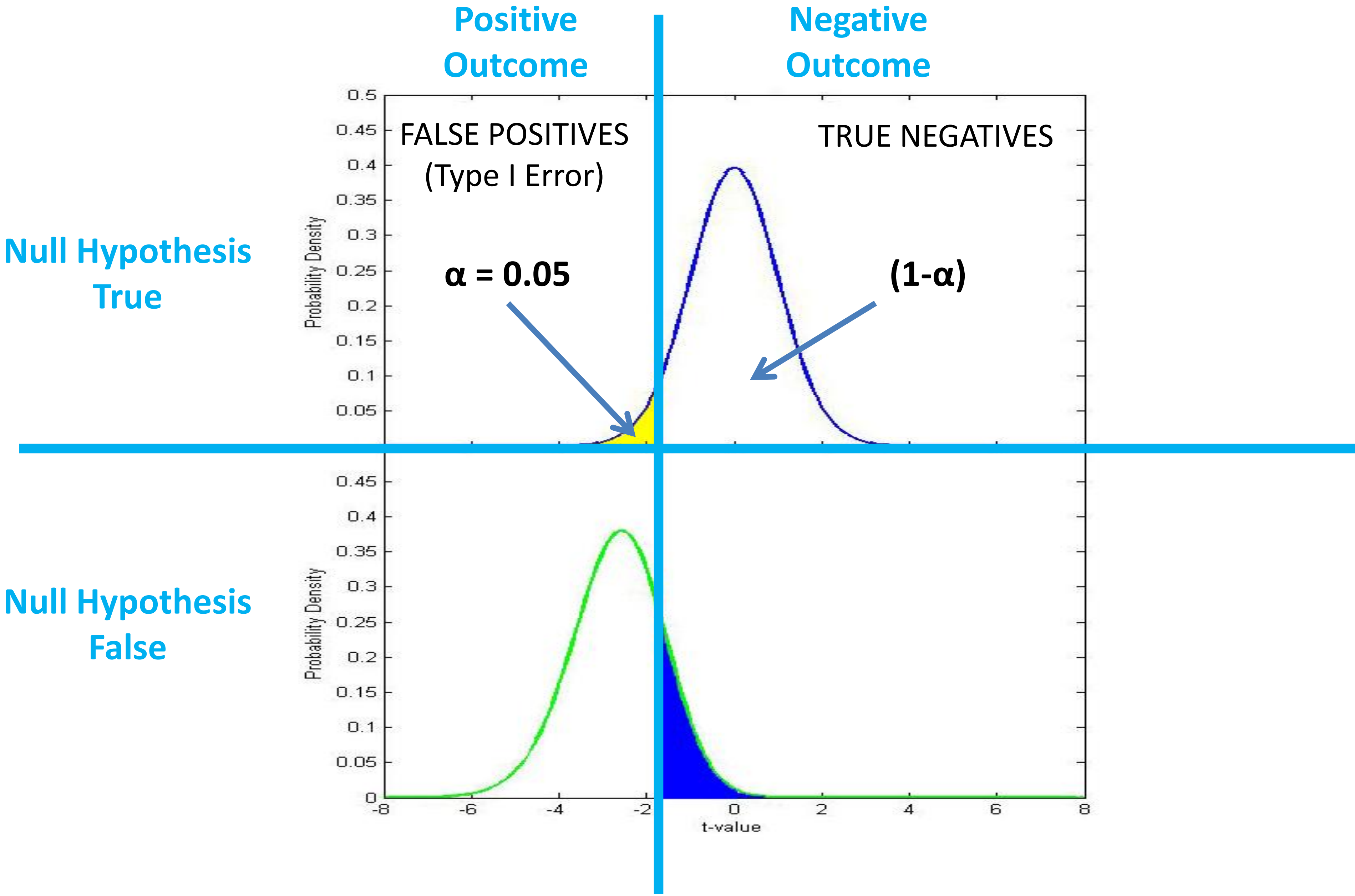
Signal Detection Analysis



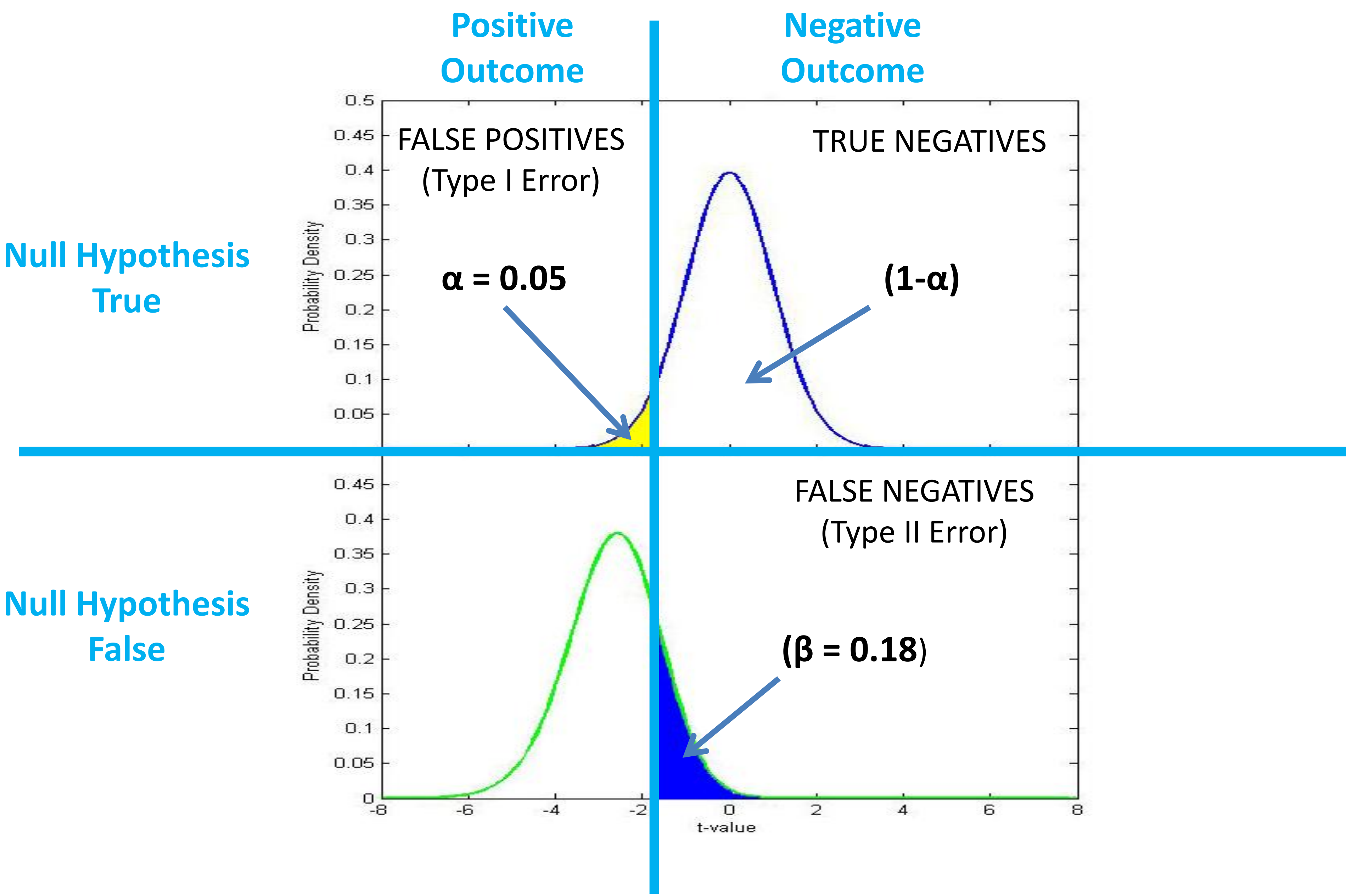
Signal Detection Analysis



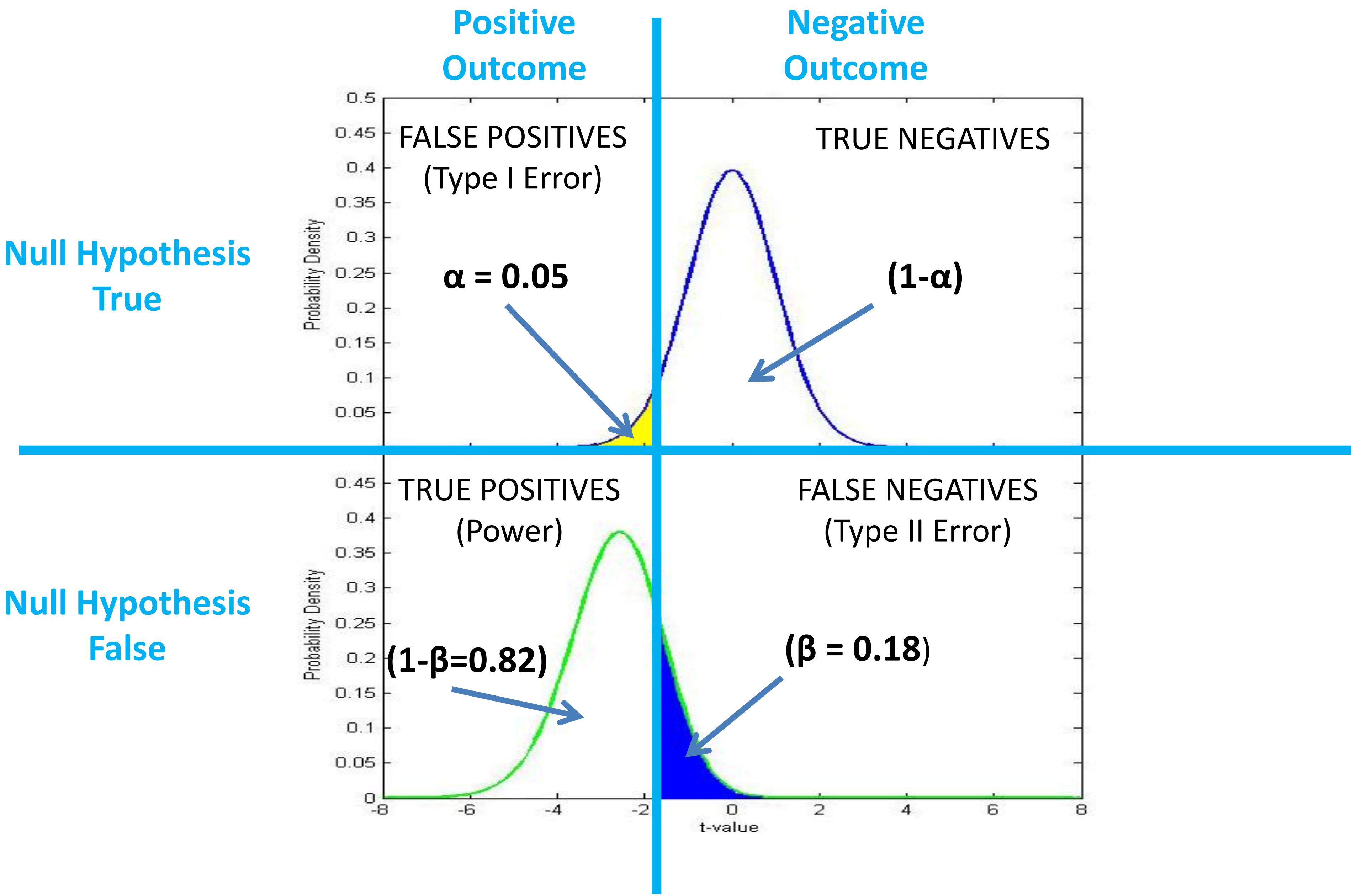
Signal Detection Analysis



Signal Detection Analysis

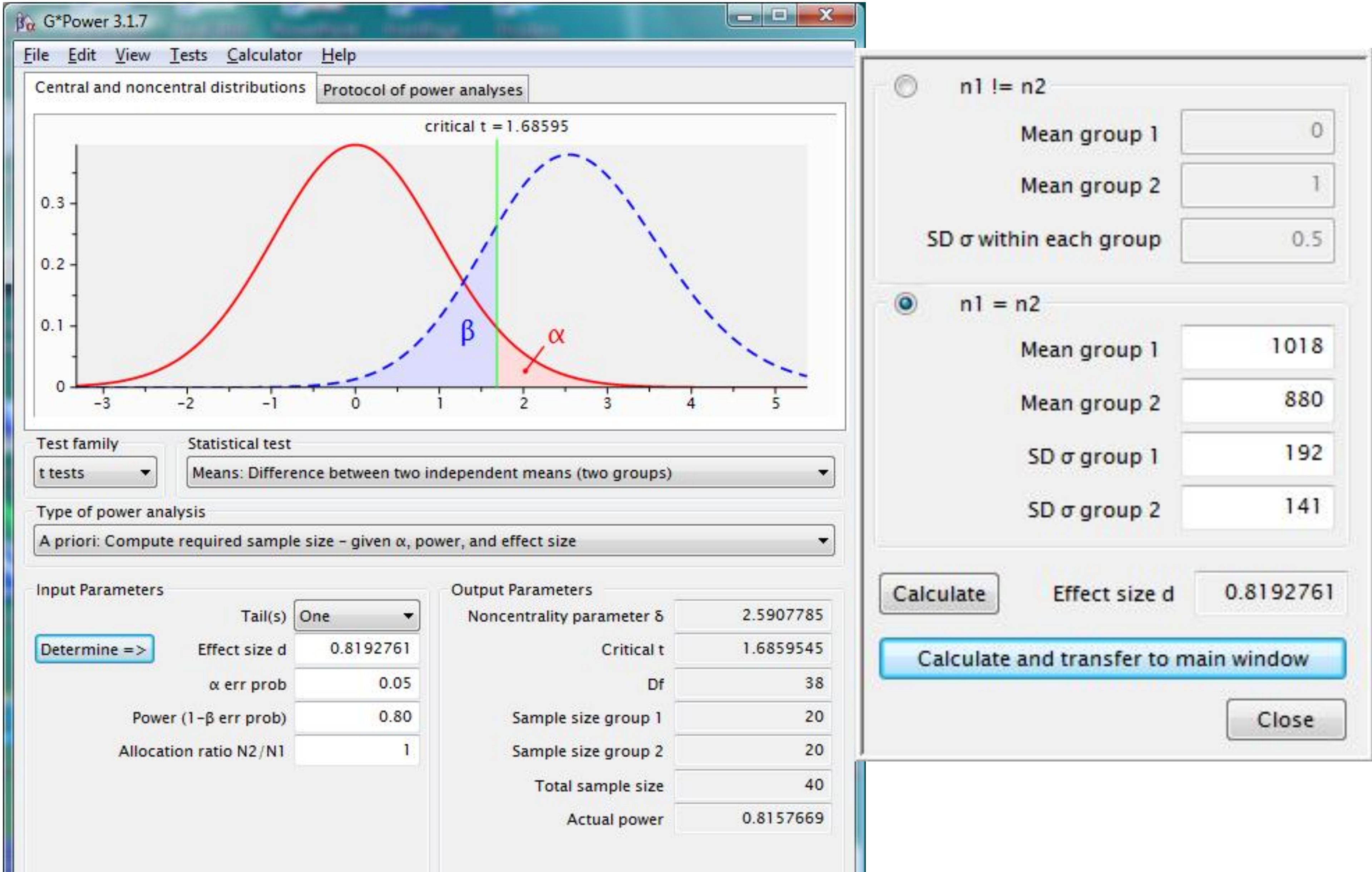


Signal Detection Analysis



G*Power Results

Nt=40; Cohen’s d=0.8; power=0.82



Computational Approach